Dijlah Journal of Engineering Science (DJES) Vol. 3, No. 1, March, 2026, pp. 353-364
ISSN: Printed: **3078-9656, Online: 3078-9664**, paper ID: 137

Deriving Effective Differential Equations from Temporal Graph Data Using Inverse Methods

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Article Info

Article history:

Received Jan, 27, 2026
Revised Feb., 25, 2026
Accepted Mar., 15, 2026

Keywords:

Temporal graphs
Inverse modeling
Differential equations
Graph neural networks,
Physics-informed machine learning

ABSTRACT

The temporal graph data offers an effective model of complex time-dependent systems across various systems such as traffic networks, brain connectivity, weather forecasting, epidemiology and physical systems. But there is still a real challenge of closing the divide between the discrete-time observations of networks and continuous-time dynamical models. It is a systematic review of the approaches to extracting effective differential equations to the data of temporal graphs by means of inverse modeling, focusing on rigorous mathematical backgrounds. We divide the existing methods into such families as graph neural differential equations, continuous-time representation learning, PDE discovery frameworks, operator learning methods, and physics-informed universal differential systems. The theoretical matters covered in the review are identifiability, stability, scalability, and robustness, with practical issues being model validation and cross-domain applicability. Inverse methods have been applied to solve problems in traffic, neuroscience, weather, epidemiology, and groundwater systems to demonstrate how the inventive methods can be used to reconstruct interpretable and physically significant governing equations. The synthesis gives a full conceptual and mathematical view of the temporal graph based inverse modeling and sets the basis of future studies in data discovery of complex dynamical systems.

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1. INTRODUCTION

The temporal network and the dynamics of such networks have been of intense interest in the recent years since complex systems in fields like neuroscience, traffic flow, epidemiology and climatology science are increasingly basing their study on time-resolved, relational data. Temporal graphs give a mathematically organised description of such systems, in which nodes represent entities and the interactions between them are represented as edges that change with time. It is important to understand the continuous-time dynamics of such networks, not only to predictive modeling but also to analyze theoretical understanding of them. In spite of the increasing amount of temporal data, the gap between discrete network data and the process of constructing interpretable continuous dynamical models is wide. Although the classical graph theoretic methods emphasize the topological measures and network statistics, they are sometimes unable to describe the underlying dynamics of the network in terms of the underlying differential equations. This gap inspires the desire of mathematically rigorous synthesis of a systematic connection between discrete time graphs and continuous differential equations. The recent surveys highlight the extent and variety of methods of this new interdisciplinary field. Liu et al. (2025) gave a detailed overview of graph-ODE frameworks, their theoretical and practical applications to the matter of spatio-temporal systems [1]. The combination of partial differential equations with the model of deep learning was highlighted by Huang et al. (2025)

[2], and the authors showed that operator-based and physics-informed inference can be used to infer governing equations on complex data sets. original contributions in physics-informed machine learning [3] laid the groundwork of instilling physical laws known into neural structures with the promise of stability and interpretability of the learned behavior. Hao et al. (2022) further expounded convergence, quantification of uncertainty and generalization characteristics of physics-informed models in multi scale systems[4]. In the meantime, examined data-driven discovery of PDEs, which shows the theoretical basis of inverse modeling and in which the governing equations can be reliably derived based on observed data[5] . Taken together, these papers find a dire necessity of a unifying mathematical framework that is not only capable of connecting discrete time graphs with continuous time systems of differentiation, but must also offer theoretically justified methods of inverse modeling, operator learning and physics informed inference. This current review seeks to overview these developments, and especially the manner in which mathematical foundations are relevant to deriving effective differential equations using temporal graph data, and the various methodological families, theoretical issues and applications of time-based graph data. Through this, it tries to create a unified conceptual framework that can guide further research as well as practice in the various fields of science.

2. Material and Method

2.1 Mathematical Foundations: Graphs, Dynamics, and Differential Equations

In this section that the formal mathematical basis is laid down to the reconstruction of effective differential equations based on the temporal graph information. It deals with two fundamental aspects including modeling of time graphs as mathematical objects, and describing dynamical systems in continuous-time with respect to these changing network structures. The discussion is based on recent theoretical and applied papers that explicitly represent graph representations together with the standard use of differential equations modeling.

2.1 Temporal Graph Representation

The temporal graphs give a natural mathematical representation of the system where interaction between entities changes with time. In its formal definition, a temporal graph may be defined as.

$$G(t) = (V, E(t), A(t)) \tag{1}$$

where $V = \{v_1, \dots, v_N\}$ denotes the set of nodes, E(t) represents the time-dependent edge set, and $A(t) \in R^{N\times N}$ is the corresponding temporal adjacency matrix. This formulation allows the evolving relational structure of the system to be represented as a continuous or piecewise-continuous function of time.

More recent results of Gravina et al. (2024) makes temporal graphs formal with irregularly sampled time series, and defines Temporal Graph Ordinary Differential Equations (TG-ODEs), where the state of nodes, as well as the connectivity, can change continuously with time [6]. In their formulation, the authors stress the importance of considering the graph as a dynamic entity and not a fixed host of time-series data. Equally, Eliasof et al. (2024) thoroughly examine the theoretical domain of the time variable in the context of a graph neural network based on differentiation equations, showing that the concepts of time discretization and sampling rate have a primary impact on the mathematical soundness of the graph model in the continuous time setting [7]. Regarding the applied spatio-temporal point of view, Zhou et al. (2023) introduce a spatial temporal dynamic graph differential equation network where the adjacency structure is adapted over time [8]. Their framework highlights how temporal graphs can be represented through time-indexed operators, often expressed as

$$A(t_k), k = 1, \dots, T, \tag{2}$$

for discrete observation times, while still supporting the reconstruction of an underlying continuous dynamical process.

All taken together, these papers prove the concept of temporal graphs being not just a type of data structure, but a mathematically well-established evolving system, which is the required precondition of developing inverse problems that can lead to the finding of governing differential equations.

2.2 Dynamical Systems on Graphs

After a temporal graph has been given a formal definition, we move to the task of describing the dynamics using this changing structure. One of the simplest mathematical models is a continuous-time dynamical system on the nodes of the graph, pinned down to a coupled system of ordinary differential equations:

$$\frac{d_{xi}(t)}{dt} = f(x_i(t)) + \sum_{j=1}^{N} A_{ij}(t)\, g(x_i(t), x_j(t))\,, i = 1, \ldots, N, \tag{3}$$

Where $x_i(t)$ denotes the state of node i, f(·) represents intrinsic node dynamics, and g(x_i)encodes interaction effects mediated by the graph structure.

This expression has been widely utilized in the case of spatio-temporal prediction and networked dynamics modeling. By way of example, Fang et al. (2021) propose spatio-temporal graph ODE networks of traffic flow prediction, in which graph node states evolve on a continuous-time neural differential equation subject to the graph Laplacian [9]. Their paper shows that the translation of discrete-time graph convolutions in continuous ODE formulas provides a more theoretical model of time evolution. Based on this view, Han and Wang (2024) suggest an adaptive graph convolution neural differential equation that involves the joint learning of the node dynamics along with the graph structure in a continuous-time formulation[10]. They use the model that explicitly parameterizes the system as a neural representation of the differential equation.

$$\frac{dX(t)}{dt} = F_\theta(X(t), A(t)), \tag{4}$$

where X(t) is the matrix of node states and F_θ is a learnable function constrained by graph topology. This formulation reinforces the interpretation of graph-based learning models as instances of data-driven differential equation discovery.

More recently, Shi and Zhang (2025) build upon this model by suggesting the use of dynamic graph neural different equations to learn continuous spatial-temporal evolution of traffic systems [11]. In their work, it is stressed that continuous-time formulation is not a convenient modeling tool, but a mathematical framework that is required to describe a long-term dependence, stability and smoothness properties of the underlying physics. Combined with contributions, these lead to a strict theoretical correlation between a temporal graph representation and a continuous theory of dynamical systems. The mathematical rationale of the inverse modeling goal of this review is necessary because it demonstrates that the idea of temporal graphs information as discrete measurements of an underlying system described by effective differential equations holds true. Figure 1 shows this conceptual framework, and the pipeline in the methodology between temporal graphs and recovered differential equations and their applications.

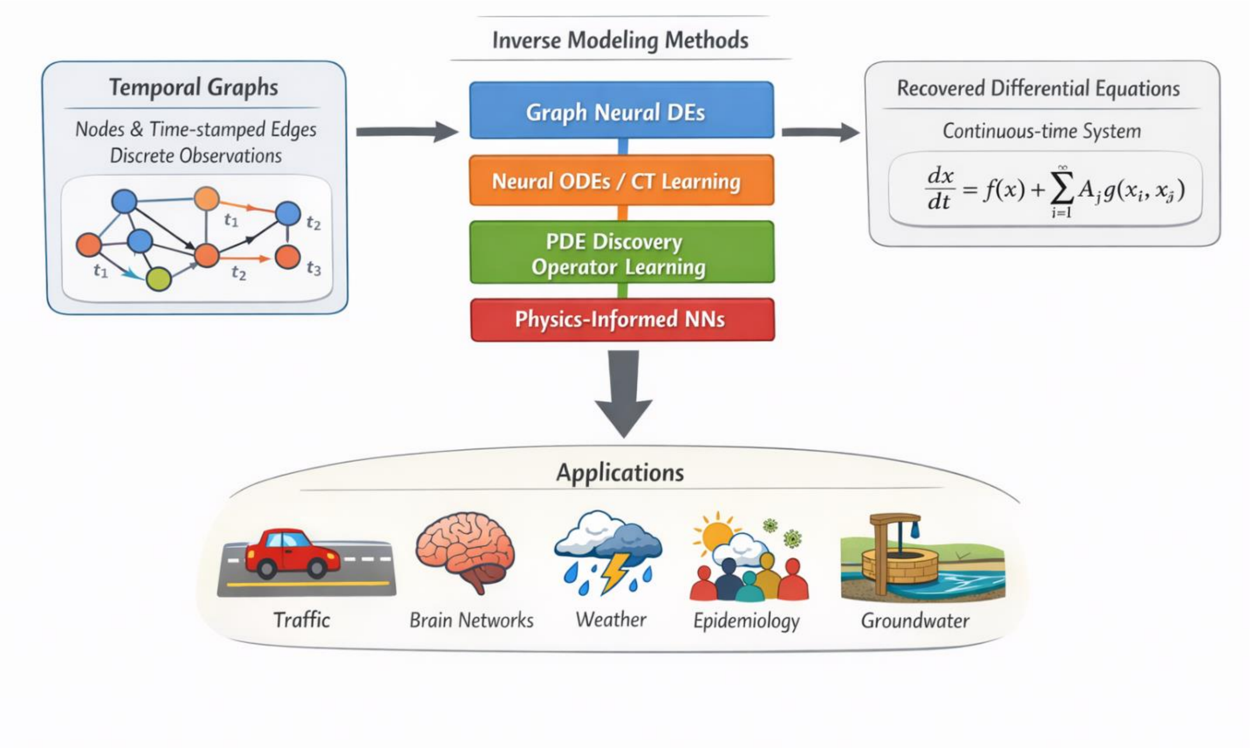

Figure 1: Idea outlook of inverse modeling of temporal graphs to the equations of continuous differentiation and applications. The figure shows how discrete time graphs are transformed into inverse modeling and generate recovered continuous differential equations and their practical implementation in traffic, brain, weather, epidemiology and groundwater systems.

2.3 The Inverse Problem: Formulating the Problem

One of the goals of this review is to pose the task of deriving effective differential equations of a time graph as a well-defined inverse problem. Unlike forward modeling, in an inverse problem the law governing equations of a system are unknown and the system trajectories are determined directly through measurements. Suppose the data of the temporal graphs under observation are modeled by node state trajectories at a discrete time:

$$X(t_k) = [x_1(t_k), x_2(t_k), \dots, x_N(t_k)], k = 1, \dots, T. \quad (5)$$

The underlying assumption is that these observations are generated by an unknown continuous-time dynamical system of the form:

$$\frac{dX(t)}{dt} = F(X(t), A(t), \theta), \quad (6)$$

where F(·) denotes the governing dynamics, A(t) is the temporal adjacency structure, and θ represents unknown parameters. The inverse problem consists of identifying the functional form of F, the parameters θ, or even the structure of the equation itself from data.

Tanyu et al. (2023) provide a thorough mathematical analysis of the inverse problems related to data-driven modeling and state that the latter is usually ill-posed in the Hadamard sense, and the problem is characterized by non-uniqueness and instability [12]. These are especially severe in the highly dimensional spatio-temporal systems, where only a few observations have to be used to recover the complex governing equations. Individually, Depina et al. (2022) [13] define the inverse problem as the minimization of a composite loss:

$$L = L_{data} + \lambda L_{physics}, \quad (7)$$

whereL_{data}enforces consistency with observed data and $L_{physics}$,encodes the residual of the differential equation. This formulation highlights how inverse modeling can be grounded in classical variational principles.

Maslyaev et al. (2021)[14] offer a framework of reconstructing partial differential equations starting directly at spatio-temporal data to find sparse symbolic structures that can most effectively explain the observations as viewed through the lens of equation discovery. Their effort makes one believe that nothing less than parameter estimation is the inverse problem, but rather the process of recovering the mathematical form of the governing equations. Rackauckas et al. (2020) [15] present a more broad-based formulation in the form of universal differential equations, where known mechanistic components are used together with trainable function approximators:

$$\frac{dx}{dt} = f(x,t) + u_\theta(x,t), \quad (8)$$

where u_θ is a universal approximator (e.g., neural network) learned from data. This framework provides a mathematically flexible bridge between classical differential equation theory and modern data-driven inference.

Together, these formulations establish a rigorous conceptual foundation: deriving effective differential equations from temporal graph data can be understood as a structured inverse problem, governed by identifiability constraints, stability considerations, and theoretical limitations inherent to inverse modeling.

2.4 Families of Inverse Methods of Deriving Governing Equations

This part is a review of and a classification of the main methodological families that have developed to rebuild governing differential equations using data. Instead of introducing these methods as a new contribution, the emphasis here is on integration of the available mathematical paradigms and the theoretical basis of the same. Graph Neural Differential Equations

4.1. The form of Graph Neural

Differential Equations (GNDEs) can be viewed as one of the most straightforward efforts to combine graph structure with time with continuous-time dynamical modeling. The dynamics of the node states in this paradigm are given as:

$$\frac{dx(t)}{dt} = F_\theta(X(t), A(t)), \quad (9)$$

where $F_\theta$is a graph-structured function, often parameterized using neural networks constrained by the adjacency matrix.

Temporal Graph ODEs which uses irregularly sampled data explicitly modelling the dynamics of the nodes and the changing graph structure as continuous time dynamics. Their construction shows time graphs can be viewed, themselves, as dynamical systems and not as the time series support systems. Liu et al. (2025) synthesize in detail a wide range of models incorporating the use of differential equations and graph neural networks, and survey this field. Their work emphasizes the fact that GNDEs extrapolate discrete graph convolutional architectures to continuous-time operator learning frameworks. A number of applied schemes also demonstrate the mathematical consistency of this scheme. Traffic dynamics described by the spatio-temporal graph ODE networks, whereas adaptive graph presented convolution neural differential equations, where the both the dynamics and the connectivity is learnt jointly. Likewise, Shi and Zhang (2025) observe that theory smoothness and stability of spatio-temporal learning is enhanced by continuity. Regarding the neuroscientific community, Tang et al. (2024) allow showing that ODE-based graph embeddings can provide interpretable representations of the useful brain connectivity [16]. Wang et al. (2025) are more recent to introduce causal graph convolution to neural differential equations, which further supports the concept that GNDEs can be used to not only make predictions but also causal and mechanistic interpretations [17]. Continuous-time Representation Learning (Beyond Graphs) The paper's fourth section emphasizes the idea of continuous-time representation learning (Beyond Graphs) that extends graph-based representation learning techniques to continuous-time systems

2.5 Continuous-time Representation Learning (Beyond Graphs)

The fourth section of the paper focuses on the concept of continuous-time representation learning (Beyond Graphs) which extends graph-based representation learning methods to continuous-time systems. In addition to explicitly graph-structured systems, neural ordinary differential equations are a generalization of discrete observations of continuous-time dynamical systems. A canonical formulation is:

$$\frac{dh(t)}{dt} = f_\theta(h(t), t), \tag{10}$$

Where $h(t)$ is a latent state learned from data.

Liang et al. (2021) use this framework on the concept of trajectory modeling explaining that Neural ODEs are an alternative to discrete recurrent architectures that is principled [18]. They use their work to reinforce the larger thesis that a large number of systems that are time-dependent can be better considered in the continuous than in the discrete form.

Li et al. (2020) focus on scalable gradient estimation of stochastic differential equations, offering mathematical frameworks that can be used to train continuous-time models efficiently, which is based on a theoretical optimization viewpoint [19]. This funding of contributions to mathematical plausibility of continuous-time learning as a basis of inverse dynamical reconstruction.

2.6 PDE Learning and Discovery of equations

An almost parallel approach to methodology is explicitly interested in finding partial differential equations through data. They are especially applicable when there are temporal graph data that are discretizations of spatially continuous systems.

Iakovlev et al. (2020) present the idea that one can apply the graph neural networks to learn the dynamics of continuous-time PDEs with a sparse set of observations, which is equivalent to the graph being considered to be a discretization of a spatial domain[20]. This literature offers a conceptual gap connecting the graph-based learning and classical numerical analysis.

Zhang et al. (2020) suggest physics-informed neural networks to learn stochastic PDEs in modal space, where they show that modes of data-driven methods can be used to recycle meaningful differential operators instead of just making accurate predictions[21]. Accordingly, Maslyaev et al. (2021) present the EPDE framework with a focus on symbolic discovery of PDE structures.

The same treatment is once again offered by Tanyu et al. (2023), who categorize the equation discovery methods by their mathematical assumptions, identifiability, and stability guarantees. All these studies together tend to suggest that inverse modelling can not only reconstruct parameters but also find the structural form of governing differential equations.

2.7 Approaches to Operator Learning

The methods of operator learning are relevant to equation discovery and seek to learn functions between spaces of functions, instead of learning specific trajectories. Fourier Neural Operator of Li et al. (2020) [22] assumes that parametric PDEs can be modeled as:

$$G: a(x) \mapsto u(x), \tag{11}$$

where G is learned directly from data using spectral representations.

Based on this paradigm, Li et al. (2022) suggest transformer-based architectures that can be used to learn PDE operator, which again supports the view of neural models as approximations of infinite-dimensional operators [23]. More recently, Liu et al. (2024) present a multi-resolution framework which retains PDE structure during learning in a very explicit way and shows that operator learning can be trained to obey known mathematical properties of the underlying system [24].

Such methods apply in particular in the case of temporal graph models, where the task can be to not only predict trajectories but also to find a way to approximate the operator under which the graph states are propagated forward in time.

2.8 Physics-Informed and Univeral Differentiated Frames

Physics-informed learning models offer a general methodological philosophy of inverse modeling. In general, the general theory of physics-informed machine learning described as governing equations serve as inductive biases, which restrict the learning process and increase identifiability.

Hao et al. (2022) provide a general methodological overview and divide physics-informed methods based on the type of problems, mathematical formulation, and area of application. They make it clear in their work that inverse modeling should be respectful of the mathematical structure of underlying physical laws in order to obtain reliable reconstruction.

Rackauckas et al. (2020) introduce a concept of universal differential equations that gives a notably flexible formulation, where known terms of a differential equation can be extended with trainable terms. It is interpretable mathematically and has the power to model data expressively. Lastly, Shukla et al. (2022) also present scalable algorithms to physics-informed neural and graph networks that showed that mathematically constrained inverse modeling could be applied to a large-scale system [25]. In Figure 2, a comparative schematic of the assumptions, data requirements, stability, and scalability of the principal types of inverse modeling methods, the Graph Neural Differential Equations, Neural ODEs / Continuous-Time Learning, PDE/Operator Learning, and Physics-Informed universal Differential Equations (PINNs) are presented.

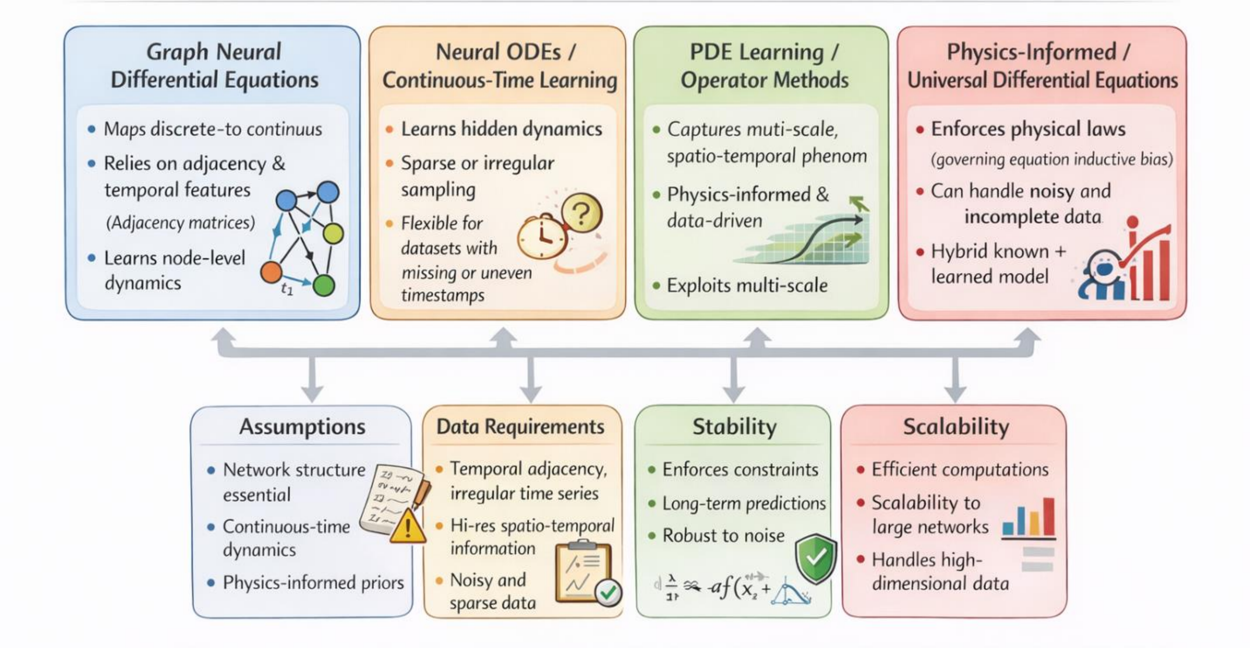

Figure 2: Comparative schematic of temporal graph data inverse modelling methods: assumptions, data requirements, stability, scalability and mathematical properties.

2.9 Bridging Discrete Temporal Graphs and Continuous Differential Systems

A central theoretical challenge in reconstructing effective differential equations from temporal graph data lies in reconciling the discrete nature of observations with the assumption of underlying continuous-time dynamics. In most applications, node states and graph structures are observed at discrete time instants $t_1, t_2, \ldots, t_T$, while the true system is assumed to evolve according to a continuous dynamical law. Formally, if the system is governed by

$$\frac{dX(t)}{dt} = F\big(X(t), A(t)\big), \tag{12}$$

then the available data consist only of samples X(t_K), which necessitates approximation, interpolation, and numerical reconstruction.

Eliasof et al. (2024) present a theoretical analysis of this problem and show that the temporal discretization of the graph neural differential equation model may have a significant impact on the well-posedness and generalization . They demonstrate in their work that sampling frequency introduces implicit regularization on the reconstructed dynamics, and that a wrong definition of discretization may result in models that are approximations of discrete time maps, as opposed to actual continuous time systems.

In modeling, Gravina et al. (2024) are able to handle the issue of irregularly sampled temporal graphs by formulating Temporal Graph ODEs directly where continuous latent system trajectories are estimated between observation points. They focus on the fact that interpolation is much more than a numerical convenience, but is a component of the mathematical model, and that the reconstructed dynamics can be viewed as approximations to smooth underlying differential equations.

Neural ordinary differential equations models offer a more general mathematical computational account of the interpolation between discrete data and continuous dynamics. Liang et al. (2021) show that latent trajectories trained using neural ODEs can be interpreted as a solution to parameterized differential equations, despite being trained using only discrete sampling of the observations. This is in line with the theoretical assumption that discrete time graph data is amenable to being viewed as noisy samples of a continuous trajectory.

Based on the perspective of the inverse problem theory, Tanyu et al. (2023) highlight that discretization and numerical differentiation cause further ill-posedness when discovering equations. Even small errors in the sampled data can result in large differences in the reconstructed derivatives and as such, regularization and model constraints are important. Their review confirms the hypothesis that the problem of closing the gap between discrete observations and continuous differential equations is not merely a question of computational complexity, but a question of basic mathematical nature that deals with approximation theory, stability and consistency.

A combination of these results confirms that theoretical justification of the reconstruction of continuous-time differential equations using temporal graph data is delicate, and has to be handled principledly to manage discretization, sampling, and approximation errors.

6. Strategies of validation and mathematical evaluation

In addition to reconstruction, another factor in inverse modeling that is vital is the mathematic validation of derived differential equations. The term validation used here is not just simply a reference to predictive accuracy but to consistency, stability and robustness of the dynamical system that was reconstructed.

Tanyu et al. (2023) present one of these perspectives when stating that the equation discovery approaches ought to be judged on their identifiability and consistency features. Particularly, a reconstructed model ought to approach the actual governing equation as both the quality and the amount of data improves, a feature that is directly connected with the statistical consistency notion of an inverse problem.

Another parameter that is required is robustness to noise. On the example of physics-informed inverse problems, Depina et al. (2022) prove that physically constrained loss functions can be used to achieve a substantial increase in stability to measurement noise. Their formulation allows the solution to be rebuilt to satisfy the differential equation in some weak sense hence offering a mathematically based validation criterion not just by minimizing error.

Causal and structural consistency are also important to be considered when validating in the spatio-temporal systems. Xia et al. (2023) introduce a causal analytical design of performing the evaluation of spatio-temporal graph forecasting models, it is important to note that proper forecasting is not sufficient to support the assertion that the learned dynamics are a natural representation of the underlying mechanisms [26]. Their effort emphasizes the need to check the validity of reconstructed dependencies and interactions as being structurally meaningful which can be directly applied to the aim of obtaining interpretable differential equations.

Collectively, these views imply that the establishment of inverse modeling methods must be based on mathematical concepts of stability, consistency, identifiability, and structural fidelity as opposed to being guided by empirical performance measures solely.

2.10 Comparative Theoretical Analysis of Methods

Considering the variety of inverse modeling methods, there is a need to do a comparative theoretical analysis of the various methods to get an insight into the strengths and weaknesses thereof. This review focuses on comparison on mathematically significant basis rather than simply assessing methods based on empirical standards and, in particular, identifiability, stability, scalability, and assumptions.

Liu et al. (2025) offer a detailed literature review of graph-based differential equation models and classify them by architecture assumption and theoretical characteristics. They find in their survey that continuous-time graph models are better expressed in terms of identifiability guarantees, but can be unstable to optimization.

In a more general view, Huang et al. (2025) assess the combination of the mathematical basis of differential equations and deep learning models and point out that most of the recent methods emphasize computational efficiency over mathematical consistency. They suggest that the way of developing methodologies in the future should be concerned with the inclusion of restrictions ensuring well-posedness and theoretical interpretability.

Likewise, Hao et al. (2022) offer a systematic description of the physics-informed machine learning approaches, which are categorized as forward, inverse, and equation discovery. They point out in their analysis that various families of methods implicitly base themselves on various mathematical assumptions, especially on the smoothness, observability and structure of the solution space.

Lastly, Brunton and Kutz (2024) note that the end objective of discovering data-driven differential equations is determined by its capacity to recover parsimonious and interpretable models. They claim that sparse representation-based, operator-theory based and dynamical systems theory-based methods have a higher likelihood to lead to scientifically significant governing equations than black-box approaches.

This comparative analysis shows that the field is not just methodologically diverse, but also there is a theoretical fragmentation. One of the main donations of this review is thus to supply a mathematically based synthesis that will explicate the connections between these strategies as well as point out theoretically acceptable standards upon which future evolution must proceed.

3. Results and Discussion

Evidence of Mathematical Reconstruction in the Application Domains

Invoking of temporal graph data to continuous differential equations has been applied to many real-world systems and has been empirically and mathematically validated in these systems.

Traffic Systems

Graph differential equations have been used to derive continuous-time dynamics of transportation networks based on discrete sequence measurements. Fang et al. (2021) came up with a spatial-temporal graph ODE network to predict traffic flows, illustrating that the graph structure and the temporal change could be strict encoded in the form of differential equations. Shi and Zhang (2025) went one step further and used dynamic graph neural differential equations to simulate continuous traffic dynamics, which treats stability and interpretability in the re-created ODEs . These findings were strengthened by Zhou et al. (2023) who incorporated time graph structures in dynamic equations to predict traffic in the city, which shows that inverse approaches can be used to understand both local and global network interactions.

Brain Networks

Tang et al. (2024) have used the spatio-temporal graph ODE models on the structural networks of the brain to reconstruct the dynamics of effective connectivity using timely resolved neural measures. Their methodology demonstrates how inverse differential modeling can be able to draw readable continuous-time representations of the complex biological networks that support discrete neuroimaging measurements with mechanistic dynamics.

Weather Systems

Weather prediction has also been implemented with hierarchical spatio-temporal graph models. Ma et al. (2023) applied graph-based neural differential equations to simulate multi-scale atmospheric processes and demonstrated that the continuous differential representations that are learnt based on discrete meteorological data could retain the underlying physical structures and are capable of predicting well [27].

Epidemiology and Mobility

Wang and Yamamoto (2020) utilized partial differential equations in epidemiology models based on mobile data to forecast the dynamics of COVID-19 at the regional level [28]. The paper shows how the temporal graph-informed PDEs can be used to recover patterns of disease spread using discrete data regarding human mobility, and constitute a mathematically rigorous model of data-driven disease epidemic forecasting.

Physical (Groundwater) Systems

Depina et al. (2022) demonstrated the simulation of unsaturated groundwater flow processes with the help of physics-informed neural networks. They verified the correctness of the continuous differential equations derived by the inverse modeling of the governing PDEs in case of sparse or noisy input data by encoding the governing PDEs directly into the inverse modeling framework.

Taken together, these applications indicate that temporal graph data can be inversely modeled to produce mathematically consistent, interpretable and physically relevant differential equations that can be used to connect discontinuous observations with continuous dynamical systems in a wide range of fields.

Theoretical Problems and Unsolved Problems in Mathematics

Although much progress has been made in this regard, there are still a number of theoretical problems which remain in deriving useful differential equations using temporal graph data.

Identifiability and Ill-Posedness

One of the main issues is identifiability: in discrete time, there may be no information about whether the estimated diffusion of a system is unique to the actual system. The stochastic character of multivariate PDEs provided by Bolin and Wallin (2020) showed that the ill-posedness occurs when the space of solutions is not partially restricted by observations [29]. Nadeem, et al. (2021) also quantumified identifiability of inverse problems with sparse or noisy measurement displaying that sparse or noisy measurements can considerably undermine the fidelity and distinctiveness of the inferred governing equations[30].

Model Uncertainty and Stochasticity

In real world systems, inverse modeling is complicated by stochastic perturbation as well as imprecision in measurements. Brunton and Kutz (2024) stressed that the discovery of the differential equation based on data should provide the explicit consideration of noise and uncertainty to prevent overfitting and have a physical interpretation. Karniadakis et al. (2021) also reported that physics-informed machine learning methods, although restrictive to known physical laws, do not necessarily solve stochasticity and non-uniqueness problems, unsolved, in the theoretical basis of the inverse methods.

To overcome these difficulties, there is a need to formulate methods that place identifiability restrictions, strongly address stochasticity and quantify uncertainty in recovered equations. In addition, a formal stability, convergence, and interpretability guarantee is an outstanding mathematical issue of interest to the expanded uptake of inverse differential modeling in temporal graph systems.

Towards a Coherent Mathematical Relaxation

The variety of inverse algorithms to find useful differential equations based on the temporal graph data encourages the search of a common approach to mathematics. The aim of such a framework is to combine discrete temporal graph representations, continuous time differential modeling with operator-based learning into a self-consistent formalism.

Liu et al. (2025) offered a conceptual synthesis that grouped the existing graph-based differential models based on their assumptions, representational power, and scalability with a primary focus on a theoretical framework that allows comparing identifiability, stability, and expressiveness of methods to each other. This perspective was continued by Huang et al., (2025) who provided a systematic review of the integration of differential equation constraints into deep learning designs with the emphasis on the mathematical similarities of seemingly unrelated methods.

Computationally, the universal differential equation framework by Rackauckas et al. (2020) offers a formal framework of implementing learnable neural components into a mechanistic framework, a differential isotope, which aligns purely data-driven methods with physics-constrained methods. Hao et al. (2022) added to this synthesis by showing that physics-informed operator learning and neural PDE frameworks can be seen as specialized versions of a general functional-analytic model, which proves to be a rigorous justification of the cross-domain applicability.

Collectively, these works contribute to the development of the mathematically based unified set of approaches according to which discrete time time series based on time graphs, continuous dynamical systems, operator learning, and physics-informed constraints are considered in one and the same formalism. This framework is not only effective in explaining the theoretical basis of existing approaches, but also helps in making successful comparisons and integrating the new approaches when it comes to inverse modeling.

3 CONCLUSION

This review has critically examined the recent developments in procedures of acquiring viable differential equations in circumstances where the data is a temporal graph and emphasis was on the theoretical foundation and the mathematical soundness of the notion of inverse modeling. Beginning with the official account of the temporal graphs, we highlighted the concept of how discrete-time network can be reliably projected to continuous time dynamical systems that provide a coherent mathematical base of the research of the developing networks.

The other problem addressed in the review was the formalization of the problem of inversion, an awareness of the conditions in which time-stamped observations can be taken, where governing equations and governing parameters and the coupling functions can be determined reliably. Those were some common methods that were classified together due to their aptness for solving inverse problems, including: Graph-based neuronal differential equations, Continuous-time representation learning, PDE discovery frameworks, Operator learning, Physics-Informed Universal Systems. We explained the theoretical assumptions, identifiability, stability, and robustness of each of the families and gave examples by illustrating mathematical principles in which applicability and weakness of each theoretical framework were established.

The convergence, approximation, and operator-theoretic consistency of discrete time data to continuous differential models became an important subject and the theme of bridging discrete time data to continuous differential models. The diversity of application fields such trafficking networks and brain connection, weather prediction, epidemiology and groundwater flow has demonstrated that, mathematically rigorous methods of inverse could be applied to construct interpretable and physically significant groups of governing equations in high-dimensional and complex data.

The review has also reported that lack of resolved theoretical questions included ill-posedness, the stochasticity, observability (partially) and multi-scale dynamics, in which theory ought to offer structures in which the inferred models should be unique, stable and generalizable. The arguments of the discussion in Section 10 resulting in a one-topic mathematical framework provides a conceptual map on how one can combine discrete graph observations, continuous dynamical models and operator-based learning into a single formalism increasing the interpretability and cross-domain applicability.

Overall, the review presents the mathematical usefulness of inverse modeling methods of the graphical time-data: the methods provide a formal relation between discrete measurements and continuous dynamical systems, enable them to generate governing equations at the operator level, and offer theoretical principles of future algorithmic and applicational developments. The information acquired in this paper is an excellent precursor to the subsequent study that will need to consolidate, generalize and provide a mathematical formulation of the data-driven reverse engineering of the complex temporal networks.

Future Research Directions

In spite of a very high level of progress, there are a number of unanswered questions that could inform the further research of this interdisciplinary field.

The creation of scalable and mathematically interpretable algorithms to large-scale temporal graphs is one of the potential avenues. Brunton and Kutz (2024) note that the possibility to find economical, physically interpretable governing equations of high-dimensional data is a central issue, especially in the case of balancing between model complexity, identifiability, and stability.

As Huang et al. (2025) note, it has been observed that there is a strong need to have methods where multi-scale dynamics, uncertainty quantification and operator learning are all rigorously incorporated into a single framework where discovered differential equations are both consistent with data and the laws of physics. Liu et al. (2025) also note that the convergence of discrete graph dynamics to continuous PDE models is a catalyst that can enable the cross-domain problem of neuroscience, epidemiology and climate modeling, and finally the guarantees of converting the theory are based on convergence, robustness and interpretability.

The stochastic and partially observed systems including those where noise or incomplete measurements worsen identifiability and ill-posedness should also be considered in future studies. Formal mathematical constraints, sparsity-promoting algorithms, and physics-aware regularization have also been necessary in order to make sure that the inverse methods are generating reliable as well as generalizable governing equations.

To recap it all, the future line of research ought to focus on:

1. Generalize unified frameworks to stochastic systems of multi-scale.
2. Devise theoretically ensured, interpretable and strong inverse modeling techniques.
3. Extend them to new fields with mathematical and operator-level consistency.

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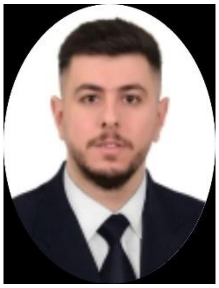	**Assistant Lecturer Wisam R. Dawood** holds a Master's degree in Pure Mathematics from India (2019). He has been a faculty member in the Department of Computer Science, College of Science, Al-Nahrain University, Iraq, since 2023. He can be contacted via email at wisam.rafid@nahrainuniv.edu.iq