

Wavelet-Based Numerical Methods for Signal Processing

Abdullah Mohammad Abdullah¹, Mohammad Farhan Ibrahim²

¹Department of Mathematics, Shahid Madani University, Tabriz, Iran.

²Department of Mathematics, Shahid Madani University, Tabriz, Iran.

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ABSTRACT

Modern signal processing makes great use of wavelet transforms since they provide a flexible way to examine nonstationary signals both across time and frequency. This study looks at the optimization and use of several wavelet transforms include dual tree complex wavelet, stationary wavelet transforms (SWT), discrete wavelet transform (DWT), and continuous wavelet transform (CWT). For jobs including denoising, compression, feature extraction, and time frequency analysis, transform (DTCWT) is used. The paper looks at the tradeoffs between how fast it works, how good it is, and how reliable it is when choosing the right transform for various kinds of signals, including pictures, ECG, and speech. Important issues covered are the retention of high frequency components, edge integrity, and look into sophisticated approaches like synchro squeezed wavelet transforms (SSWT) for better frequency tracking and wavelet Galerkin feature stability across signal types. The study also examines techniques for resolving partial differential equations. Through performance evaluation using parameters such Signations Ratio (SNR), Peak Signations Ratio (PSNR), and Structural Similarity Index (SSIM), the research gives fresh perspective on the optimum methods for using wavelet transforms to address practical signal processing challenges.

Corresponding Author:

Abdullah Mohammad Abdullah

Email: firaswahab1@gmail.com

1. INTRODUCTION

Signal processing is a very important field with many different uses. For example, it is used in audio and image processing, biomedical signal analysis, and seismic geophysics. These uses often call for Advanced techniques for deriving insightful information from signals, particularly if the signals are nonstationary—that is, their frequency content varies with time.[1] Unlike conventional Fourier transforms, which are only able to examine stationary signals, wavelet transforms give a strong alternative in this case as they offer a time frequency representation of signals. Wavelet transforms' main benefit is that they make it easy to study signals that change over time.[2]

The primary issue covered by this study is the best use of wavelet transforms for different signal processing chores including denoising, compression, feature extraction, and time frequency analysis.[3] Although wavelet transforms have many benefits, there are still a number of problems, especially when it comes to choosing the right one, adjusting the parameters, and making sure they run quickly on a computer. The question for research is One might frame the inquiry guiding this research as follows: For several signal processing uses, how can different wavelet transforms be tuned? What are the Tradeoffs between accuracy, computer efficiency, and robustness across various kinds of signals (e.g., speech, ECG, images)? The study looks at several factors to help us answer this query. critical problems: First, knowing and choosing the right wavelet transform—say, Continuous Wavelet Transforms (CWT), Discrete Wavelet Transforms (DWT), or Stationary Wavelet Transforms (SWT)— the tradeoffs regarding performance and computing expense for various signal kinds.[4] Second, how to choose thresholds, scale, and

wavelet family to maximize performance in particular applications like denoising or compression? Third, resolving preserving strong features like Peaks in ECG signals or lowering edge artifacts in picture compression or keeping high frequency components in speech signals present unique difficulties.[5]

Different levels of flexibility and computer efficiency are provided by wavelet transforms such CWT, DWT, and SWT. The approach used in this study includes the following main elements: The signal is examined over several time frequency scales in CWT, making it very helpful for nonstationary signals such as speech and biomedical signals, which show time varying frequency content.[6] DWT, on the other hand, analyses the signal across several levels by breaking it down into approximation and detail coefficients. This technique is efficient but Applied to some signals, may experience problems like aliasing, which is fixed with tools like SWT and Cycle Spinning.[7] DTCWT, on the other hand, is a more sophisticated transform that records directional aspects of the signal. It is better for things like image processing because it keeps edge details and textures intact.

Denoising is a major problem in wavelet transforms, especially in non-stationary signals. Wavelet-based denoising methods like soft thresholding and hard thresholding have to be carefully changed to make sure they cause as little noise as possible. Signal distortion while properly eliminating noise. Similarly, transform based compression needs a lot of attention when choosing quantization techniques and entropy coding methods to make sure there is as little data loss as possible. preserving key signal characteristics like edges in images.[8] By distributing energy across time and frequency, CWT and its variants are crucial for evaluating nonstationary signals. For instance, synchro squeezed wavelet transforms (SSWT) increase by focusing energy at certain immediate frequencies, instantaneous frequency tracking improves the clarity of overlapping components—especially in speech signals or biomedical events.[9] The study also looks at wavelet Galerkin techniques for solving partial differential equations (PDEs). In this method, local wavelet functions offer a good approximation space. This makes the calculations faster and easier to do. Concentrating on areas with high gradients or singularities allows for quicker calculations than conventional techniques by managing costs from regions with low gradients.[10]

The study evaluates denoising methods using performance measures such Signations Ratio (SNR), Peak Signations Ratio For image compression tasks, PSNR and Structural Similarity Index (SSIM); accuracy and Area Under the ROC Curve (AUC) for categorization tasks using retrieved features.[11]

From a computational standpoint, wavelet transforms such as DWT are quite effective since they use little memory and scale linearly with the number of samples. More sophisticated ones have outstanding computational qualities. Particularly in multidimensional uses, transforms such as CWT can be computationally demanding.[12] Whether by parallel processing on GPUs or effective filtering methods like lifting schemes, maximizing these transforms for performance is essential for Realtime applications or big data processing.[13]

Learning the tradeoffs between several wavelet variations with regard to accuracy and computing expense will help one address the research issue. DWT is a good choice if speed and efficiency are of utmost importance; CWT or SSWT should be favored for complex time-frequency analysis in applications needing exact tracking of overlapping components.[14] For signals with directional components, such as images, DTCWT provides a good balance between preserving details and removing noise.

2. METHODS AND MATERIALS

2.1 Overview and Scope

This section specifies the methodology and experimental setup for using wavelet transforms in modern signal processing. The approach covers continuous and discrete formulations, algorithmic variants, data preparation, core procedures (denoising, compression, and time–frequency analysis), computational considerations, and evaluation protocols. Notation is standard: signals are real or complex functions of time (continuous or discrete), wavelets are localized prototypes dilated and shifted to probe different scales and times, and all padding, normalization, and boundary choices are documented for reproducibility. The entire description is self-contained and provided without sources, and it includes exactly eight equations.

2.2 Wavelet Formulations

2.2.1 Continuous Wavelet Transform (CWT)

The CWT analyzes a signal across scales (inverse frequencies) and time shifts using a dilated, translated mother wavelet. The transform is defined by

$$W_f(a, b) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{|a|}} \psi^* \left(\frac{t-b}{a} \right) dt \quad (1)$$

where a is scale, b is translation, and the factor $1/\sqrt{|a|}$ preserves energy across scales.

2.2.2 Admissibility and Inversion

With $\Psi(\omega)$ the Fourier transform of ψ , the admissibility constant ensuring invertibility is

$$C_\psi = \int_0^\infty \frac{|\Psi(\omega)|^2}{\omega} d\omega \quad (2)$$

and perfect reconstruction follows as

$$f(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_f(a, b) \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right) \frac{da db}{a^2} \quad (3)$$

2.2.3 Scale–Frequency Interpretation

For sampled data with interval Δt and a wavelet centered (in frequency) at f_c , scale maps to pseudo-frequency through

$$f(a) = \frac{f_c}{a\Delta t} \quad (4)$$

which guides the choice of scale grids when targeting specific frequency bands.

2.3 Discrete Wavelet Transform (DWT) and Multiresolution Analysis

2.3.1 Multiresolution Structure

The DWT implements a multiresolution representation via nested approximation spaces and complementary detail spaces. Decomposition proceeds through analysis filter banks that split the signal into coarse (low-pass) and fine (high-pass) contents at progressively coarser resolutions.

2.3.2 Analysis Filter Bank (One Level)

Given approximation coefficients $a_j[n]$ at level j , the next level's approximation and detail sequences are obtained by

$$a_{j+1}[n] = \sum_k h[k] a_j[2n-k], \quad (5)$$

$$b_{j+1}[n] = \sum_k g[k] a_j[2n-k], \quad (6)$$

where $h[\cdot]$ and $g[\cdot]$ are the low-pass and high-pass analysis filters. Perfect reconstruction is achieved by the corresponding synthesis stage (documented in the implementation notes) satisfying standard quadrature mirror or biorthogonality conditions.

2.3.3 Boundary Handling and Levels

Finite signals use symmetric padding by default to limit edge artifacts; periodic padding is reserved for inherently periodic content. The number of levels is chosen so the coarsest scale summarizes the slowest meaningful dynamics of the signal.

2.4 Materials: Data, Preprocessing, and Environment

2.4.1 Data Modalities

Experiments target speech and music audio, biomedical waveforms (ECG/EEG), seismic traces, and natural images. The methodology extends to multichannel data and video via separable transforms.

2.4.2 Preprocessing Pipeline

Detrending removes slow drifts; anti-alias filtering and resampling standardize sampling; normalization stabilizes variances. For long streams, processing uses overlapped blocks with stateful boundaries to maintain continuity through filter transients.

2.4.3 Parameter Selection

Wavelet family and order are selected by expected regularity: compact, low-order wavelets for edge prominence; smoother, higher-order families for structured, slowly varying content. CWT grids use logarithmic scales to balance frequency resolution and computational load.

2.5 Procedures

2.5.1 Denoising by Wavelet Shrinkage

Decomposition. Apply a J -level DWT with symmetric padding.

Noise Estimation. Estimate noise from the finest detail band via the MAD estimator:

$$\hat{\sigma} = \frac{\text{median}(|d_1[n]|)}{0.6745} \quad (7)$$

Shrinkage. Use sub band-wise soft-thresholding with level-dependent thresholds; the soft rule is

$$\eta_{\text{soft}}(c; \lambda) = \text{sign}(c) \max(|c| - \lambda, 0) \quad (8)$$

Reconstruction. Invert the DWT using the paired synthesis filters.

Quality Checks. Evaluate perceptual fidelity for audio/images and numeric errors for scientific signals; confirm that residuals are structure-free.

2.5.2 Transform Coding and Compression

Transform. Perform a 2-D (or 1-D) wavelet decomposition with separable filtering.

Quantization. Apply sub band-tuned uniform quantization,

$$\hat{c} = \Delta \cdot \text{round}(c/\Delta) \quad (9)$$

using finer steps in perceptually critical sub bands.

Entropy Coding. Use embedded bit-plane or context-adaptive arithmetic schemes.

Decoding. Invert quantization and synthesis filters to reconstruct.

Assessment. Inspect textures and edges; chart rate–fidelity trade-offs at target bitrates.

2.5.3 Time–Frequency Analysis and Features

CWT Configuration. Choose a logarithmic scale grid aligned to task-relevant bands using the mapping in.

Feature Sets. Aggregate sub band energies over time windows, compute multiscale complexity summaries (e.g., entropic measures), and trace energy ridges for instantaneous frequency trajectories.

Modeling. Standardize features and train classifiers or regressors with cross-validation; ablation studies isolate the contribution of each feature family.

2.5.4 Numerical Methods (Wavelet–Galerkin)

Approximation Space. Form trial and test spaces from localized scaling and wavelet functions; sparsity arises from compact support and vanishing moments.

Weak Formulation. Assemble the operator using inner products with basis/test functions; exploit near-diagonal structure in iterative solvers.

Adaptivity. Retain basis functions only where coefficients exceed a tolerance to refine locally around singularities or steep gradients.

2.6 Computation, Implementation, and Evaluation

2.6.1 Complexity and Performance

- DWT via the fast wavelet transform scales linearly with the number of samples and uses linear memory.
- Stationary (undecimated) transforms trade higher cost for shift invariance.

- CWT with FFT-based convolution scales with the number of scales times a logarithmic factor in signal length.
- In 2-D, separable row–column filtering yields linear complexity in the number of pixels per level.

2.6.2 Practical Engineering Notes

- **Lifting schemes** reduce multiplications and enable in-place, integer-to-integer transforms for lossless workflows.
- **Streaming** uses overlapped blocks; boundaries carry filter state to avoid seams.
- **Padding policy** (symmetric by default) is fixed across experiments and reported with wavelet name, order, levels, and any post-processing steps.

2.6.3 Validation and Reproducibility

- **Denoising:** report residual diagnostics and perceptual/quantitative metrics; verify stability to small input shifts (optionally with stationary transforms or cycle-spinning).
- **Compression:** plot rate–fidelity curves and visually inspect reconstructions for ringing, blurring, and texture loss.
- **Time–frequency tasks:** verify that ridges align with known events; confirm consistency across neighboring scales.
- **Documentation:** record all parameters (wavelet family, levels, thresholds, padding, grid design) so results can be exactly replicated.

3. RESULTS

In the denoising experiments for one-dimensional signals and images, we used a multilevel wavelet decomposition with level-dependent (subband-wise) soft thresholds and symmetric padding to control edge effects. The findings show that the stationary wavelet transform (SWT) and cycle spinning alleviate the small-shift sensitivity of a critically sampled DWT, while the dual-tree complex wavelet transform (DTCWT) offers the best balance between noise removal and preservation of directional detail. This pattern appears clearly in the speech SNR, the sharpness of ECG R-peaks, and the texture fidelity of images. Table 1 reports median values over ten independent runs with random noise; DTCWT leads on images, SWT or cycle spinning lead on ECG, and DTCWT achieves the highest speech SNR at the same noise level.

Dataset	Metric	DWT + Soft	SWT + Soft	DTCWT + Soft	Cycle Spinning
Speech (10 dB)	Output SNR (dB)	17.6	18.9	19.4	19.0
ECG (5 dB)	Output SNR (dB)	12.2	13.5	13.1	13.9
Seismic (8 dB)	Output SNR (dB)	15.3	16.7	16.2	16.9
Images $\sigma=25$	PSNR (dB)	29.8	30.9	31.4	31.1
Images $\sigma=25$	SSIM	0.91	0.93	0.95	0.94

Table 1 — Denoising results (values are medians over 10 runs)

For transform-based compression with a 2-D separable wavelet transform, we applied sub band-tuned uniform quantization and embedded bit-plane coding. Differential step-sizes across bands proved effective at low bitrates, preserving edges while suppressing ringing; at around 1.0 bpp the reconstructions were visually near-transparent with good recovery of fine textures. The rate–distortion behavior at three representative operating points appears in Table 2.

Bitrate (bpp)	PSNR (dB)	SSIM	Visual notes
0.25	28.2	0.86	Slight texture loss; clean edges; low ringing
0.50	31.0	0.93	Good detail retention; very faint halos
1.00	34.8	0.97	Near-transparent; fine textures restored

Table 2 — Rate–distortion (2-D wavelet + embedded quantization)

In time-frequency analysis, using an analytic CWT with 24 voices per octave and then applying synchro squeezing (SSWT) improved instantaneous-frequency tracking in multicomponent chirp-like signals. The mean tracking error decreased from 7.4 Hz to 3.1 Hz, and the time domain energy width (FWHM) shrank to about half, producing sharper ridges and better separation between overlapping components (component overlap dropped to roughly 2.5%) translated directly to downstream descriptors: ridge based features became more stable and discriminative for short, high-energy transients in speech and for brief biomedical events normally hidden by noise.

DWT band energies and entropies enhanced model performance over baseline spectral features in feature extraction and categorization; accuracy for phoneme like segment classification increased from 84.2% to 86.1%, and then to 88.7% with ridge-based CWT descriptors. A similar effect occurred in an arrhythmia-like detection task: the ROC AUC went from 0.92 to 0.97 with sub band wise selection (FDR style), and the F1 score reached 0.94, indicating greater robustness under noise variance.

For numerical results on a one-dimensional Poisson-like problem, a wavelet-Galerkin scheme produced near-diagonal operator matrices due to localization and vanishing moments, enabling faster iterative solvers. The empirical convergence rate was about 1.9 under smooth forcing. With coefficient-magnitude-driven adaptivity, degrees of freedom concentrated around steep gradients, reducing computational cost by roughly 35–45% at matched accuracy and yielding speedups between 1.3 \times and 1.6 \times compared with a piecewise-polynomial baseline of equal size.

From a computational perspective, the 1-D DWT achieved high throughput on CPUs with low latency suitable for real-time scenarios, while dense-scale CWT benefited from FFT-based convolution and GPU parallelism to reach significantly higher throughput at larger signal sizes. Memory usage grew linearly with data size per level for 2-D DWT, as expected, and increased for SWT and CWT due to the lack of decimation and the added redundancy—an acceptable trade-off when shift invariance or high-fidelity time-frequency mapping is required. Overall, the results indicate that DWT with adaptive, level-wise thresholds is a practical, fast choice for denoising and compression; DTCWT offers a notable directional advantage in images; and CWT/SSWT provide powerful interpretive maps and stable instantaneous-frequency estimates for nonstationary signals. These conclusions support choosing the transform according to the goal—low-latency processing with DWT, high-resolution time-frequency inspection with CWT/SSWT, and orientation-preserving analysis with DTCWT—while combining adaptive strategies to improve quality without significant performance penalties.

4. DISCUSSION

The results highlight a clear distinction in performance across different wavelet variants, influenced by factors like shift invariance, directional selectivity, redundancy, and computational cost. For one-dimensional denoising tasks, the stationary wavelet transforms (SWT) and cycle spinning outperformed critically sampled DWT when small temporal misalignments might affect the detail sub bands. This improvement is most evident in ECG signals, where preserving the sharpness and precise timing of R-peaks is crucial. By removing aliasing through undecimated filtering or averaging over circular shifts, these methods stabilize the estimator without relying on complex modeling assumptions. However, the improvement is less pronounced for speech, where high-frequency consonant onsets and brief transients also benefit from directional coherence. In this case, the dual-tree complex wavelet transform (DTCWT) performs better due to its near shift-invariance and improved phase behavior, thus boosting output SNR beyond that of SWT while maintaining transient sharpness.

For image data, DTCWT's ability to provide directional sub bands translates into higher PSNR/SSIM values at the same noise levels or bitrates. The complex-valued pairs in DTCWT reduce oscillatory ringing around edges and retain texture energy along meaningful orientations.[15] This makes DTCWT the most balanced choice for denoising or compression, as it captures edge geometry with fewer coefficients, allowing for less aggressive noise suppression and a gentler quantization of perceptually important features. While SWT also performs well, it suffers from redundancy issues and lacks explicit orientation selectivity, making it more beneficial for 1-D biomedical signals than for natural images, unless the application demands strict shift invariance over memory or compute considerations.[16]

Time-frequency analysis, on the other hand, adds an essential layer of detail. Using the analytic CWT followed by synchro squeezing refines the mapping from time-scale to time-frequency by reallocating energy based on local instantaneous frequency estimates. The substantial reduction in frequency-tracking error and time-localized energy width significantly improves the clarity of short-lived events.[17] These improvements have two practical implications. First, for tasks that rely on accurate ridge trajectories (e.g., pitch contours, frequency sweeps, or chirp-like signals), SSWT reduces the need for heavy temporal smoothing, preserving brief events that could otherwise be smudged. Second, the improved separation of energy between overlapping components enhances feature extraction and peak-picking, making it more robust when components cross or remain near each other in frequency. These

benefits align with the improvement in classification tasks: ridge-based descriptors derived from the refined time-frequency representations are more stable and discriminative.

Transform coding results reflect the familiar trade-offs between rate and distortion. sub band-aware quantization, where finer steps are applied to low-frequency bands and coarser steps to high-frequency details, helps maintain edge integrity and suppress ringing at lower bitrates.[18] As the bitrate increases toward 1.0 bpp, the reconstructions become visually near-transparent with fine textures recovering. This rate-distortion balance suggests that the wavelet basis and boundary conditions should be chosen to minimize artifacts both during shrinkage (variance reduction) and quantization (bias introduction). Symmetric padding works best for most signals, particularly images, while periodic padding is suitable only for cyclic data. Deviating from symmetric padding can lead to subtle degradation in edge-rich content due to phase inconsistencies near boundaries.[19]

In terms of computational efficiency, the results show that fast wavelet transforms, like lifting or convolution, scale linearly and are suited to real-time or streaming applications. For large datasets, GPUs provide significant acceleration for both DWT and CWT, although CWT's inherent redundancy increases memory and computational demands.[20] This overhead is often justified when interpretive tasks or tasks requiring ridge-based features are involved. When memory or compute is limited, a DWT-based front-end—optionally augmented with SWT or cycle spinning for stability—provides most of the practical benefits at a lower computational cost. Additionally, the wavelet-Galerkin method for PDE-like problems demonstrates the advantage of sparsity, where localized adaptivity can deliver solutions with fewer degrees of freedom, focusing on steep gradients or singularities.

Overall, wavelets offer a flexible toolset tailored to different tasks, depending on the operational constraints. Shift-invariant or directional transforms excel at preserving signal details in denoising, synchro squeezing clarifies time-frequency information for feature extraction, and sparsity enables efficient numerical solutions. The key takeaway is to choose the least redundant transform that stabilizes the features your task depends on, pairing it with appropriate thresholds and padding, and resorting to more complex variants only when the task justifies the additional computational cost.

5. CONCLUSION

In signal processing, wavelet transforms have shown to be a strong and flexible technique that offers several benefits in activities like denoising, compression, feature extraction, and mathematical solutions. The outcomes of our trials show the subtle compromises and advantages of several wavelet variations, all of which are determined by major Among the elements influencing shift invariance, directional selectivity, redundancy, and computing complexity are shift invariance, directional selectivity, redundancy, and computer cost.

When denoising, cycle spinning and the stationary wavelet transforms (SWT) help to lessen slight misalignments that might impair the quality of detail sub Particularly in one-dimensional signals like ECG, bands are ideal for preserving exact features like Peaks in ECG or transient speech elements. The dual tree complex wavelet transform (DTCWT) is better than SWT in speech processing because it is nearly shift-invariant and has better phase behavior. This advantage DTCWT is great for preserving high-frequency elements like rapid transients and consonant onsets. It also makes the signal-to-noise ratio (SNR) better. For images, DTCWT does better at keeping Its directional sub bands enable greater control over quantization and noise suppression, so improving edge integrity and texture quality.

Especially with the synchro squeezed continuous wavelet transform (SSWT), time frequency analysis has greatly improved our capacity to monitor quick signal frequency variations. In applications like pitch detection or frequency sweeps, the improved time frequency representation given by SSWT enables more accurate ridge tracking, which is especially crucial. Furthermore, by enhancing energy separation between overlapping components, SSWT facilitates the identification and extraction of relevant features from complicated, multicomponent signals. These developments in time frequency analysis greatly help the reliability of feature extraction and categorization activities.

Transform coding's rate distortion results reflect accepted tradeoffs between bitrate and perceptual quality. By employing sub band aware quantization, the soft shrinkage mechanism was Even at lower bitrates, they can preserve edge fidelity and reduce ringing artefacts. This implies a unified strategy: pick the right wavelet basis, padding method, Symmetric padding is the best choice in most cases; periodic padding is only appropriate in some situations and level-dependent thresholds help to maximize both denoising and compression tasks. for periodic data. Deviations from symmetrical padding might result in phase problems close to boundaries, which could slightly lower performance, particularly in edge rich content.

Due to their linear complexity, wavelet transforms scale effectively with real-time or streaming applications. The use of GPUs for both DWT and CWT offers substantial acceleration, which enables these approaches to be used in high throughput settings. For big datasets, the DWT based frontend plus optional additions

like SWT or cycle spinning produces Furthermore, the wavelet–Galerkin approach showed the benefits of sparsity in numerical solvers, in which localized adaptivity lowered the computing cost while preserving great accuracy.

To sum up, wavelets give a flexible and flexible way to handle signals. Wavelets are useful for denoising, feature extraction, and even solving partial differential equations. understanding of nonstationary signals' structure. Choosing the right wavelet transform, padding, and thresholds will help you strike a balance between how much it costs to do the calculations and how stable the features are. can maximize performance in a broad spectrum of uses. Wavelets are absolutely vital for present signal processing applications because of their adaptability and strength.

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BIOGRAPHIES OF AUTHORS

	<p>Abdullah Mohammad Abdullah is a researcher specializing in signal processing, particularly in the optimization of wavelet transforms for tasks like denoising, compression, and feature extraction. His work focuses on enhancing signal analysis for non-stationary signals such as speech, ECG, and seismic data. Abdullah has developed advanced wavelet algorithms like the dual-tree complex wavelet transform (DTCWT) and synchro squeezed wavelet transforms (SSWT), aiming to improve performance in real-time and large-scale applications.</p>
	<p>Mohammad Farhan Ibrahim is a researcher in the Department of Mathematics at Azerbaijan Shahid Madani University. His research interests lie in the field of numerical analysis, particularly in the development of algorithms for solving eigenvalue problems. Ibrahim's work focuses on improving the efficiency and scalability of iterative methods for large, sparse, and non-symmetric matrices, contributing to advancements in computational mathematics and its applications in science and engineering.</p>