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Recursive Least Squares Lattice Algorithm for Adaptive Equalization of a Linear Dispersive Communication Channel.

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ABSTRACT

ABSTRACT: Distortion is a serious undesirable phenomenon in the electrical and communication systems. It is produced in the transmitting channel and distorts the input signal with addition of noise to it. Because of its adverse impacts on the system, it is inevitable to eliminate or at least reduce its harmful effects to the minimum level by a simple and feasible technique. In this paper, we adopt the strategy of Recursive Least Square Lattice (RLSL) algorithm for adaptive transversal equalization to remove the effect of any type of distortion. The ingenuity of this research includes studying the convergence speed of the algorithm, degree of sensitivity to eigenvalue spread, and the regression coefficients for the equalizer for various channels. MATLAB software is used to perform the simulation work of this algorithm. The results indicate that the mean square errors have been reduced significantly throughout using this algorithm.

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1. INTRODUCTION

There are a proliferation of algorithms in the field of linear adaptive filters to solve the problem of distortion in communication channels. Basically, the linear adaptive filter has two main components. The first one is the filter structure which produces the desired output and the second one is a selected adaptive algorithm that set the filter parameters to produce the optimum output signal. So, it is clear that the selected algorithm is influenced to the great extent by the type of filter which is either finite impulse filters(FIR) [1]- [5] or infinite impulse response (IIR) [6]-[10]. There are mainly three adaptive algorithm namely, least mean squares(LMS) [11]-[13], recursive least squares (RLS) [14]- [16], and recursive least squares lattice(RLSL) [17]-[19] to solve equalization problem. All these adaptive applications have to be rigorously investigated before selection of appropriate methodology to remove the noise from the input signal and adjust the adaptive transversal filter (ATF) weights in in order to minimize the mean square error of the channel.

In applying an algorithm to the linear adaptive filters, the strength and weakness of that technique can be explored. In selection of the algorithm, there are many factors to taken in consideration such as computational complexity, convergence speed, and efficiency. In this paper, the RLSL or the QRD-LS algorithm is selected for adaptive filter to solve the least squares problem in a recursive way. In applying this algorithm, we study the learning

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curves of this technique for various channels, the convergence rate, degree of sensitivity to eigenvalue spread, and the regression

coefficients for the equalizer for various channels. Although there are many algorithms that can solve the least-squares problem such as RLS technique. It is well known that the lattice based algorithms are more efficient because they endorse modular implementation and needs less number of arithmetic operations [20]. The lattice recursive least-squares (LRLS) algorithms are less complexity than RLS algorithm, while on the other hand the RLSL technique is considered as very complicated in term of software implementation, but it is much less complicated than RLS in computational complexity. The RLSL algorithms generate forward and backward prediction errors simultaneously and provide the prediction and the general adaptive filter (joint-process estimation) solutions of all intermediate orders from 1 to N simultaneously. Therefore, the order of the adaptive filter can be changed without influencing the previous computations and there is no need to redesign of original system. This behavior enables the user to activate or deactivate Sections of the lattice efficiency in real time based on performance requirements. Unlike the RLS algorithm [21] that needs only time-recursive equations, the lattice RLS algorithms use time-update and order-update. RLSL algorithms performance similar to any other RLS algorithm when implemented with infinite precision arithmetic, while they perform differently with finite-precision implementation. There are different forms of the RLSL algorithm such as standard RLSL algorithm based on aposteriori errors, RLSL algorithm based on a priori errors, and algorithms with error feedback.

Solving the problems of the RLSL forward and backward prediction are necessary to obtain the order-updating equations which are intrinsic to the RLSL algorithms and the results are obtained following the same methodology of the conventional RLS algorithms. Although this algorithm is primarily used in linear adaptive filters, it has many other applications in the fields of acoustic echo estimation, noise control, and adaptive signal processing [22]- [24] because of its reduced computation complexity [25].

In this study, unknown distortion is produced by the channel and we assume that the data are all real valued. Figure (1) depicts a block diagram to study the RLSL algorithm. Random–number generator (1) generates the signal $\{a_n\}$ that employs to probe the channel and has values +1, and -1, zero mean with variance equal to 1. Random–number generator (2) generates additive white noise $\{v(n)\}$ that distorts the channel output and has a zero mean with variance $\sigma^2{}_V$ =0.0001, therefore SNR of 40 dB is obtained at the filter input. These two generators are independently working of each other. The impulse response of the channel is as follows:

$$h(n) = \begin{cases} \frac{1}{2}\left[1 + cos\left(\frac{2\pi}{\omega}(n-2)\right)\right] & for\ n = 1,2,3 \\ 0\ , & otherwise \end{cases}$$

u(n) is produced by the convolution sum of a(n) and υ (n):

$\mathbf{u}(n) = b^T a(n) + \upsilon(n)$; n=l, 2,….., N

$\mathbf{u}(n) = b_1\ a_{n-1}, + b_2\ a_{n-2} + b_3\ a_{n-3} + \upsilon(n)$; Where $a_{n-i} = 0$; $n-i < 0$.

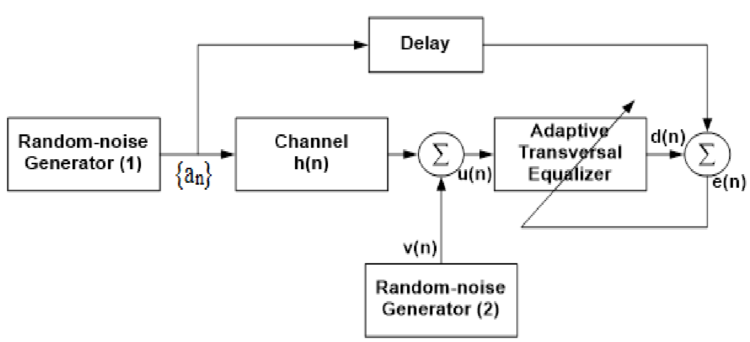

Fig.1 Block diagram of an equalizer

As mentioned elsewhere in this Section, this algorithm is designed to solve equalization problem and estimate MSE of the adaptive filter. An approximate MSE is computed as following:

$$MSE(n) = \frac{1}{K}\sum_{K=1}^{K} e^2{}_k(n);\ n=1,2,3,\ldots\ldots., N.$$

Where $e^2{}_k$ (n) is instantaneous squared error, N is the number of data samples$\{a_n\}$, and K is the number of the independent runs required for averaging.

2. THE PROPOSED RLSL ALGORITHM

Basically RLSL algorithm combines two filters, forward filter and backward filter, into one single structure and therefore this lattice is more efficient filter for generation of forward and backward prediction errors at the same time. For updating the lattice order, there is no need for redesigning the original system, and the filter order can be increased without affecting the previous computations. To update the equalizer parameters, the forward estimation error is combined with backward estimation error.

2.1 Problem Statement

In this paper, the presence of distortion in communication channel is the intrinsic problem. Therefore, it is required to remove its effects by executing RLSL algorithm The procedure to implement of this algorithm is as follows:

2.2 Initialization of Algorithm:

1. Given a filter order M and an input data sequence of length N, initialize as follows:
For m=0, 1, ……., M and n=0, 1,……, N

$$b_m(n) = 0;\ f_m(n) = 0;$$
$$B_m(n) = \delta;\ F_m(n) = \delta;$$
$$\Delta_{m-1}(n) = 0;\ \gamma_m(n) = 1;$$
$$\rho_m(n) = 0.$$

2. Starting of algorithm and filter update.
After forming the noise and the desired signals, start the RLSL algorithm as follows:
(a) Initialize the parameters of the input stage.

$$b_0(n) = u(n);\ F_0(n) = u(n);$$
$$B_0(n) = F_0(n) = \lambda\ F_0(n-1) + u^2(n)\ ;$$
$$\gamma_0(n) = 1;\ e_0\ (n)= d(n).$$

(b) Time and Order-update of lattice parameters.
For each lattice stage m= 1,2, ………, M

Time-update:

$$\Delta_{m-1}(n) = \Delta_{m-1}(n-1) + \frac{b_{m-1}(n-1)\, f_{m-1}(n)\ )}{\gamma_{m-1}(n-1)}$$
$$\Gamma_{f,m}(n) = -\frac{\Delta_{m-1}(n)}{B_{m-1}(n-1)}$$
$$\Gamma_{b,m}(n) = -\frac{\Delta_{m-1}(n)}{F_{m-1}(n-1)}$$

Order-update:

$$f_m(n) = f_{m-1}(n) + \Gamma_{f,m}(n)\, b_{m-1}(n-1)$$
$$b_m(n) = b_{m-1}(n-1) + \Gamma_{b,m}(n)\, f_{m-1}(n)$$
$$F_m(n) = F_{m-1}(n) + \Gamma_{f,m}(n)\, \Delta_{m-1}(n)$$

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$$B_m(n) = B_{m-1}(n-1) + \Gamma_{b,m}(n)\,\Delta_{m-1}(n)$$
$$\gamma_m(n) = \gamma_{m-1}(n) - \frac{b^2{}_{m-1}(n)}{B_{m-1}(n)}$$

c) Estimation

$$\rho_m(n) = \lambda\,\rho_m(n-1) + \frac{b_m(n)}{\gamma_m(n)}\,e_m(n) \qquad \text{(time-update)}$$
$$\kappa_m(n) = \frac{\rho_m(n)}{B_m(n)}$$
$$e_{m+1}(n) = e_m(n) - \kappa_m(n) b_m(n) \qquad \text{(order up-date)}$$
$$\hat{d}(n) = \sum_{i=1}^{M} K_i(n) b_i(n) \qquad \text{(output)}$$

3. After m=M, follow the same procedure until n=N.

3. METHODOLOGY

The adaptive transversal equalizer structure depicts in Figure (1), has two main parts. First part is the digital filter to eliminate all distortions and all non-desirable noises, while second part is the adequately selective adaptive algorithm that updates all equalizer parameters to obtain the desired reference signal d(n) as shown in Fig. 2. In this filter, the digital filter output y(n)= d(n)-e(n). The procedure of updating and adjusting of all filter parameters and derivation of all equations related to this filter have been shown in Sub Section 2.2 The MATLAB software is employed to carry out all simulation works of this technique.

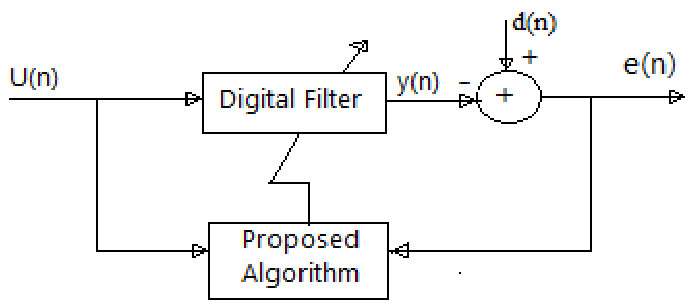

Fig. 2 Block diagram of an Adaptive Equalizer

4. DISCUSSION OF THE RESULTS

Here we will discuss the effects of selection of adaptive transversal filter(ATF) number of taps M (filter order M), and effect of eigenvalue spread on the filter performance.

1- Effect of filter order:

We have selected the ATF order M=11 due to the cost effective reason and better performance as mentioned in [21].

2- Effect of eigenvalue spread

This algorithm is used to update the tap weights for the same equalizer described previously. We use the four channels namely, w= 2.9,3.1,3.3, 3.5 in our study. The variance of the noise is selected to be $\sigma^2{}_V$ = 0.0001 and δ to be 0.01 as shown in the Table 1:

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Table 1 Parameters used in the algorithm

w	2.9	3.1	3.3	3.5
λ_{min}	0.3339	0.2136	0.1256	0.0656
λ_{max}	2.0295	2.3761	2.7263	3.0707
$\chi = \frac{\lambda_{max}}{\lambda_{min}}$	6.0782	11.1238	21.7132	46.8216

δ	0.01
No. of data samples N	$\geq$ 1000
No. of independent runs K	$\geq$ 200
SNR	40 dB

Comparing the learning curves of RLSL algorithm shown in Figure (3) below for different values of (w), it is evident that the convergence speed is almost insensitive to the eigenvalue spread. On the hand, the MSE is high for channel with higher (w). If we compare these results with that obtained with RLS algorithm [21], it is clear that both algorithms demonstrate relatively the same performance in terms of convergence speed and MSE steady state values.

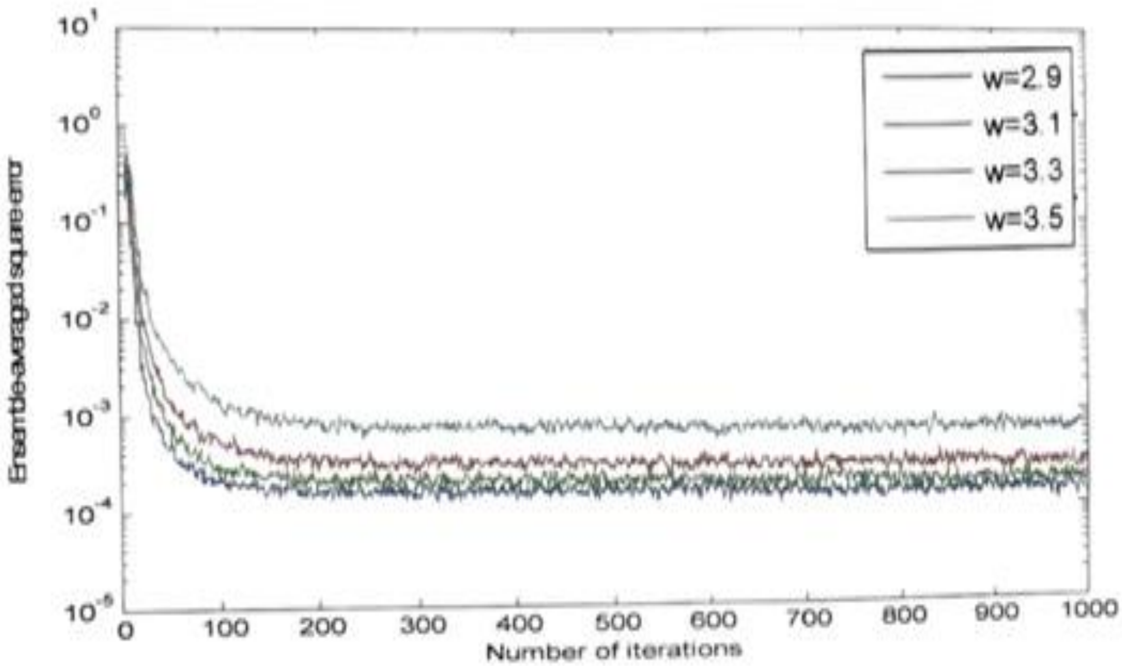

Fig.3 Learning curves of RLSL algorithm for various channe

Figure (4) shows the forward and backward reflection coefficients for various channels for the final stage as a function of iteration. It is shown that there is an initial transient behavior for both cases, but the jump is higher in case of forward. Then both curves converge to steady state value.

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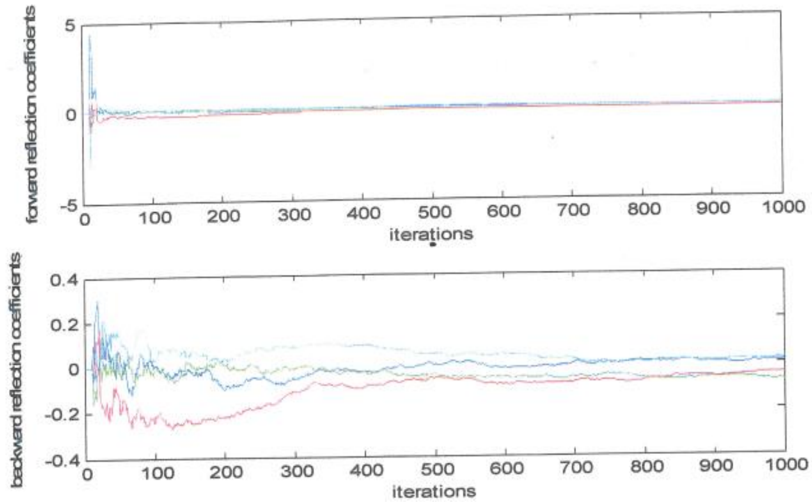

Fig.4 The ensemble average forward and backward reflection coefficients for various channel

Figure (5) demonstrates the likelihood parameter of the $\gamma_m(n)$ for the final stage as a function of iteration. It is noted that this parameter is not affected by eigenvalue spread of the filter in

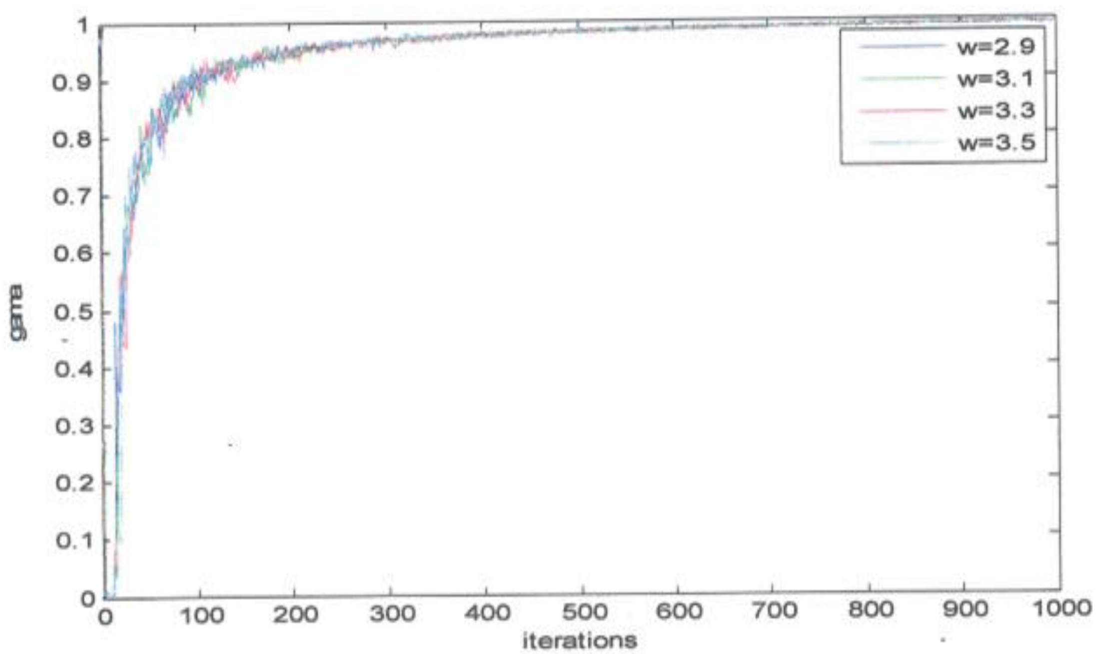

Fig.5 The ensemble average likelihood parameters for various channels

The regression coefficients for the equalizer for various channels are plotted in Figure 6. It is indicated that the regression coefficients are symmetric are symmetric around 6 with relatively equal magnitudes, but in opposite direction.

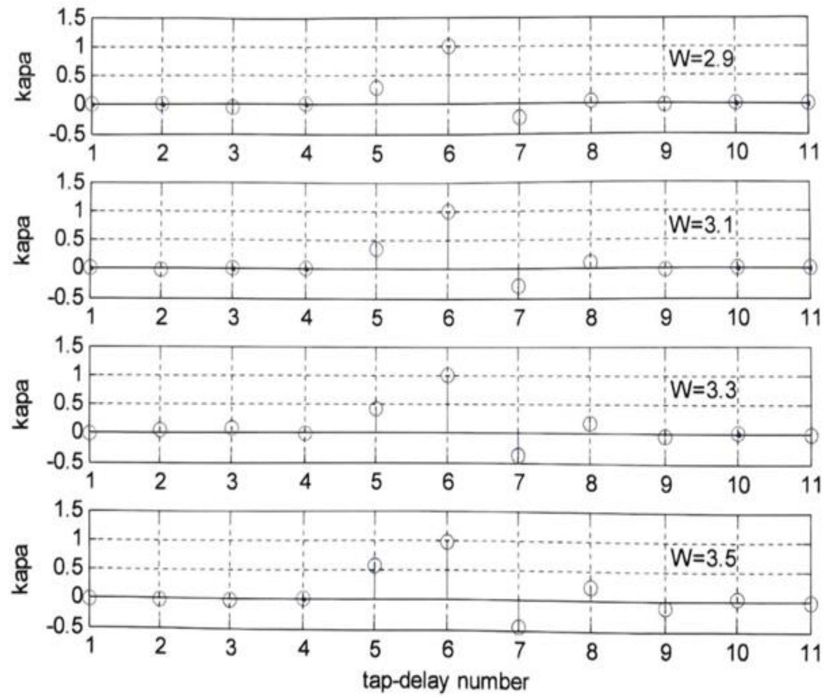

Fig.6 Regression coefficients for four channels

5. SUMMARY AND CONCLUSIONS

In this algorithm, firstly we have discussed the impact of the proper selection of low ATF order (11) in terms of feasibility and cost-effective design. Secondly, the effect of eigenvalue spread has been studied in detail and from the learning curves of RLSL strategy, it is demonstrated that the convergence speed is not affected by eigenvalue spread, but MSE is reduced significantly with reduction of eigenvalue spread and vice versa. This denotes that this algorithm is the most effective technique to reduce the adverse impacts of distortion and noise in communication channels to the minimum level. Thirdly, from the curves of ensemble average forward and backward reflection coefficients for various channels for the final stage, it is shown that there is an initial transient behavior for both cases, but the jump is higher in case of forward. In addition to that, and from the plotting of the curves of likelihood of the γ_m(n) for the final stage, it is noted that this parameter is not affected by the variations of the eigenvalue spread. Finally, we concluded that the regression coefficients are symmetric around 6 as expected with relatively equal magnitudes, but in opposite directions.

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	Nomenclatures
$e^2(n)$	Instantaneous squared error
v(n)	White noise; w Distortion parameter; χ Eigenvalue spad.
$\mathbf{e_f}$ (k, i)	Instantaneous aposteriori forward prediction error.
$\mathbf{e_b}$ (k, i)	Instantaneous aposteriori backward prediction eror.
$\Gamma_{f,m}(k,)$	Forward reflection coefficient.
$\Gamma_{b,m}(k,)$	Backward reflection coefficient.
$\mathbf{f_m}$ (k)	Forward prediction error.
b_f (k)	Backward prediction error.
a(n)	Apriori error.
	Greek Symbols
δ	Regularization factor.
λ	Forgetting factor.
	Abbreviations
ATF	Adaptive Transversal Filter
LMS	Least Mean Square
RLS	Recursive Least Squares
RLSL	Recursive Least Squares Lattice
SNR	Signal to Noise Ratio
MSE	Mean-Square Error